MATH 334 Honors Advanced Calculus	Autumn 2018

Lecture 1: September 26

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1.1 Basic Notations

$\mathbb{R}$	real numbers
(a, b)	ordered pair
$(a_1, a_2, \ldots, a_n)$	ordered list
$\langle a, b\rangle$	inner product

We use $\mathbb{R}$ to denote the real numbers.

1.2 Norm

We use $\|a\|$ to denote the norm of a, which usually means $\|a\|_2$ - the euclidean norm, defined as

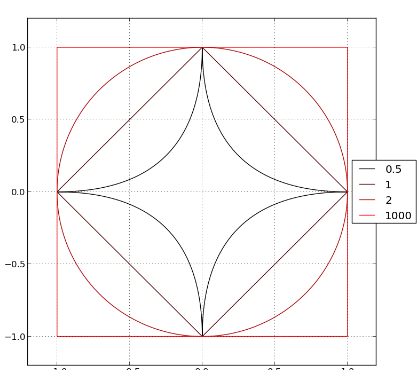
$$\|a\|_2 \equiv \langle a, a\rangle^{1/2}.$$

Here are some other norms:

$$\|a\|_1 \equiv \sum a_i$$
$$\|a\|_\infty \equiv \max\{|a_1|, |a_2|, \ldots, |a_n|\}$$

Then, we can define the unit sphere by $\|x\|_2 = 1$, and defined the unit ball by $\|x\|_2 \leq 1$.

The image below demonstrate unit circle in different norms.



Generally, the p-norm is defined as

$$\|x\|_p \equiv \left(\sum_i |x_i^p|\right)^{1/p}.$$

Notice that, there may be other norm defined, but they usually follow rules below.

1. Triangle inequality: $\|x + y\| \leq \|x\| + \|y\|$
2. $\|x\| \geq 0$
3. $\|tx\| = |t| \, \|x\|$

1.3 Inequalities

Theorem 1.1 $|\langle v, w\rangle| \leq \|v\|_2 \cdot \|w\|_2$

This is called as **Cauchy's Inequality**

Proof: See next lectures' notes. ■

Theorem 1.2 $\|v + w\|_2 \leq (\|v\|_2 + \|w\|_2)^2$

This is called **Minkowski Inequality**.

Proof:

$$\|v + w\|_2^2 \leq (\|v\|_2 + \|w\|_2)^2$$
$$\langle v + w, v + w\rangle \leq (\|v\|_2 + \|w\|_2)^2$$
$$\langle v, v\rangle + \langle w, w\rangle + 2\langle v, w\rangle \leq \langle v, v\rangle + \langle w, w\rangle + 2 \|v\|_2 \|w\|_2$$

Then by **Theorem 1.1**, we have $\|v + w\|_2 \leq (\|v\|_2 + \|w\|_2)^2$. ■